Quiz 3 #3:

$$f(x,y) = ye^{xy}$$

Tangent plane @ $(x,y) = (0,1)$.

Common mistake was incorrect differentiation, often for f_y.

$$f_y(x,y) = e^{xy} + ye^{xy}x$$

You may remember the tan. plane eqn as

$$z = z_0 + f_x \cdot (x - x_0) + f_y \cdot (y - y_0).$$

$$f_x = f_x(x_0, y_0)$$

$z = x^2 + y^2$ x, y, z are real numbers and this expresses a relationship between them.

$f(x,y) = x^2 + y^2$ f is a function $\mathbb{R}^2 \longrightarrow \mathbb{R}$ and this defines f on an arbitrary input.

Note: $f(u,v) = u^2 + v^2$ defines the exact same function f.

A mistake I saw was using $f_x(x,y)$ when supposed to use $f_x(x_0,y_0)$.

One then gets

$$z = 1 + (y^2 e^{xy})(x-0) + (e^{xy} + xye^{xy})(y-1)$$

This is definitely not a plane! Plus it's way worse to compute than $f(x,y)$ too...

Two ways of writing tan. plane eqn:

$$z = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

$$F_x(x_0,y_0,z_0)(x-x_0) + F_y(x_0,y_0,z_0)(y-y_0) + F_z(x_0,y_0,z_0)(z-z_0) = 0 \qquad (*)$$

This can be written as

$$\nabla F(\vec{r}_0) \cdot (\vec{r} - \vec{r}_0) = 0.$$

A mistake I saw: ppl used $(*)$ with $F(x,y,z) = ye^{xy}$.

Correction: since we are looking @ $z = ye^{xy}$, rearrange to get

$$\underbrace{ye^{xy} - z}_{F(x,y,z)} = 0$$

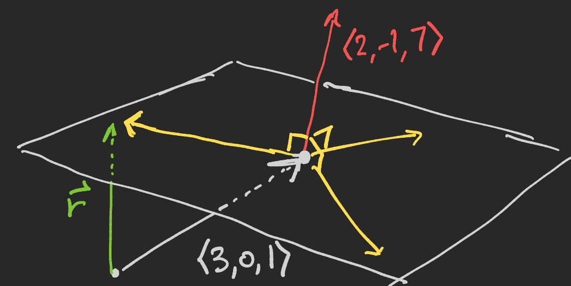
Q2 #1:

What kind of object does

$$\underline{(\vec{r} - \langle 3,0,1\rangle)} \cdot \underline{\langle 2,-1,7\rangle} = 0$$

describe?

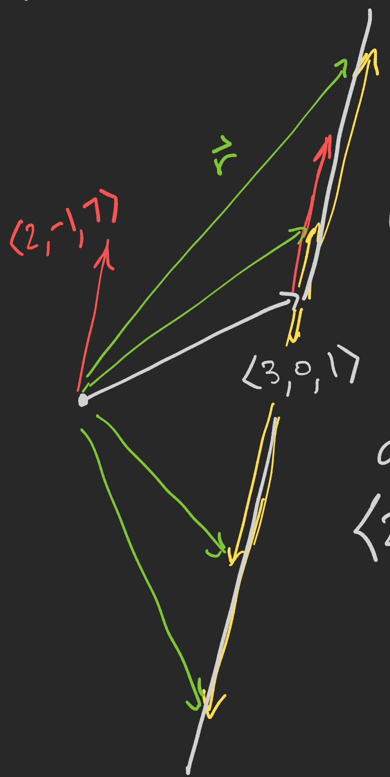
$(\vec{r} - \vec{r}_0) \cdot \vec{n} = 0$



What does

$$(\vec{r} - \langle 3,0,1\rangle) \times \langle 2,-1,7\rangle = \vec{0}$$

describe?



This is a line through $(3,0,1)$ in the direction $\langle 2,-1,7\rangle$

Does this go through $(0,0,0)$?

Sol 1: $3+2t=0$

$0+(-1)t=0$

$1+7t=0$

No solution.

Sol 2: $\langle 3,0,1\rangle$ and $\langle 2,-1,7\rangle$ are not parallel, so no.

14.6 #29

The direction of fastest change is

$$\nabla f = \langle 2x-2,\ 4y-4 \rangle$$

and we need this direction to be same as $\hat{i}+\hat{j} = \langle 1, 1 \rangle$, so we need

$$\nabla f = c\,\langle 1, 1 \rangle$$

c positive real #.

$$\begin{cases} 2x-2=c \\ 4y-4=c \end{cases}$$

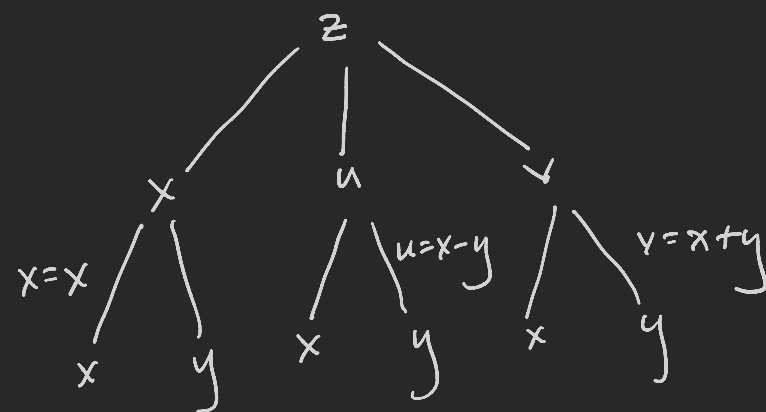
You will find infinity many solutions. (they should form a ray).

14.5 #47

$$z = \frac{1}{x}\left[f(\overset{u}{x-y}) + g(\overset{v}{x+y})\right]$$

Verify

$$\frac{\partial}{\partial x}\left(x^2 \frac{\partial z}{\partial x}\right) = x^2 \boxed{\frac{\partial^2 z}{\partial y^2}}$$



$$\frac{\partial z}{\partial y} = \frac{1}{x}\boxed{f'(u)}(-1) + \frac{1}{x}\boxed{g'(v)}(1)$$

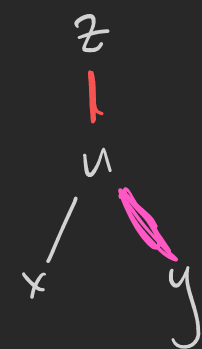
$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = (-1)\left(\frac{1}{x} f''(u)(-1)\right)$$

$$+ \frac{1}{x} g''(v)(1)$$

Keep in mind e.g. $f'(u)$ is a function of u, which depends on x and y, so need chain rule to compute
$$\frac{\partial}{\partial y}(f'(u)).$$

e.g. to compute

$$\frac{\partial}{\partial y}\Big(\overbrace{(\overbrace{x+y}^{u})^2}^{z}\Big)$$



$$= 2(x+y)(1)$$

(though of course you could've done this directly as well.)